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Friday, September 19, 2008

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Synthesis of Highly Convergent Finite Element Models for Short Wave Propagation

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EXTENDED ABSTRACT

SYNTHESIS OF HIGHLY CONVERGENT FINITE ELEMENT MODELS FOR SHORT WAVE PROPAGATION

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SUMMARY

The presented approach for reducing the phase and group errors in short wavelength pulses propagation modeling is based upon the modal error minimization. The computational model is built of alike component substructures (CS) the matrices of which are obtained by modal synthesis. The necessary modal properties of component substructures are established by solving the cumulative modal error minimization problem for a sample domain the exact modal frequencies of which are known theoretically. Modal frequencies and shapes of the component substructure are used as the design parameters for the modal error minimization problem . After the matrices of a component substructure are obtained, they can be used to form any structure higher-order elements. Earlier the approach has been demonstrated to work well in 1D case. In this work the results for 2D rectangular meshes describing elastic and/or acoustic wave propagation have been obtained. As a result, models having up to 80% of modal frequencies with an error less than 2% can be obtained by using the optimized component substructures. Though the synthesized mass matrices are non-diagonal, the obtained dynamic models are able to simulate short transient waves and wave pulses propagating in elastic or acoustic environments by using only a few nodal points per pulse length.

KEY WORDS: modal synthesis; modal error; transient wave simulation

1. INTRODUCTION

The shape of a propagating short wavelength pulse simulated in a discrete mesh is inevitably distorted if the distance traveled by the pulse comprises a large number of lengths of the pulse. As a result, the shape and duration of the simulated pulse become very different from the values expected theoretically. An important source of distortions is the phase error inherently produced by the discrete model. The errors usually are minimized by using very dense meshes, however, this makes the simulation complex and requiring huge computational resources. The modal errors of the computational domain can be regarded as the origin of phase errors. As a consequence of them, different harmonic components of waves comprising the wave pulse propagate with different velocities and produce group errors of wave propagation.

As early as in 1982 different modal frequency convergence features of dynamic models obtained by using lumped and consistent forms of mass matrices have been noticed [2]. The convergence of modal frequencies of dynamic models can be improved by using the 'generalized' form of the mass matrix obtained as a weighted superposition of lumped and consistent mass matrices, [3]. Approaches concentrating on improvement of modal convergence properties and retaining the diagonal form of the mass matrix have been presented [4-7].
Element type 99 of LSDYNA program is intended for vibration studies carried out in time domain. These models may have very large numbers of elements and may be run for relatively long durations. This is achieved by imposing strict limitations on the range of applicability, thereby simplifying the calculations: elements must be cuboid; small displacement, small strain, negligible rigid body rotation; elastic material only. The element formulation also includes single element bending and torsion modes [8].

The non-diagonal matrices obtained by modal synthesis can give better results. In 1D case they produce models having 60-80% of modal frequencies with error values less than 3%, [9]. In this work we demonstrate that the main principles of the approach presented in [9] can be applied also for 2D rectangular wave propagation models. Computational domains are being assembled of component substructures 'optimized' in order to provide minimum cumulative modal frequency errors of selected sample domains. We demonstrate that the structure of any size assembled of such component substructures has approximately the same percentage of 'close-to-exact' modal frequencies as had the sample domain. Though the synthesized mass matrices are non-diagonal, the obtained dynamic models are able to simulate transient waves by using only a few nodal points per pulse length.

2. GENERAL RELATIONS OF MODAL SYNTHESIS

Finite element models of small vibrations and waves in elastic or acoustic continua are presented by the well known semi-discrete structural dynamic equation as

$$[\mathbf{M}]\{\ddot{\mathbf{U}}\}+[\mathbf{K}]\{\mathbf{U}\}=\{\mathbf{R}(t)\}, \qquad (1)$$

where $[\mathbf{M}],[\mathbf{K}]$ - structural mass and stiffness matrices, $\{\mathbf{R}\}$ - nodal forces vector. In problems addressed in this work we assume the damping forces to be very small and omit the damping term.

The structural matrices used in (1) can be expressed by using modal synthesis relations as

$$[\mathbf{M}]=\left([\mathbf{Y}]^T\right)^{-1}[\mathbf{Y}]^{-1}; \quad [\mathbf{K}]=\left([\mathbf{Y}]^T\right)^{-1}\left[diag(\omega_1^2,\omega_2^2,...,\omega_n^2)\right][\mathbf{Y}]^{-1} \quad , \qquad (2)$$

where $\omega_1,\omega_2,...,\omega_n$ are the modal frequencies of the model of dimension $n\times n$, and $[\mathbf{Y}]=\left[\{\mathbf{y}_1\},\{\mathbf{y}_2\},...,\{\mathbf{y}_n\}\right]$ - the modal shapes. By using relations (2) desirable dynamic properties expressed in terms of known modal frequencies and modal shapes can be supplied to model (1).

3. "OPTIMUM" COMPONENT SUBSTRUCTURES

In wave propagation models large parts of computational domains can be built of alike *component substructures* (CS) consisting of identical elements. As a limit case, a CS may consist of a single element, or may be a larger domain the shape of which is geometrically similar to the shape of the element. We need to *optimally modify the spectral properties of a CS in order to produce the minimum modal frequency error of the whole structure.*

In our approach, formation of the mode set for modal synthesis is performed by solving the eigenvalue problem for the free CS *n-r* times by taking each time different generalized mass matrices as $[\mathbf{M}^e]=k_{Lj}[\mathbf{M}_L^e]+\left(1-k_{Lj}\right)[\mathbf{M}_C^e]$, *j=r+*1*,…,n,* where r – number of rigid body modes of the CS, *n*- dimension of he CS. The weight coefficient $0\le k_{Lj}\le 1$ is the parameter the value of which is to be specified. As initial value, $k_{Lj}=0.5$ could be regarded as a reasonable choice. From each j-th eigenvalue problem solution only one (i.e., the j-th) modal frequency and modal shape is selected and included into the set used later for modal synthesis. Finally the rigid body modes 1 to *r* are generated, and the mode set for modal synthesis reads as $\omega_1,\omega_2,...,\omega_n$; $[\mathbf{Y}]=\left[\{\mathbf{y}_1\},\{\mathbf{y}_2\},...,\{\mathbf{y}_n\}\right]$.

Modal frequencies and shapes are further modified by scaling them as

$$\left[diag\left(0,...,0,\ \alpha_{r+1}^{\omega}\omega_{r+1}^2,\ \alpha_{r+2}^{\omega}\omega_{r+2}^2,...,\ \alpha_{r+n}^{\omega}\omega_n^2\right)\right]=\left[diag\left(\omega^2\right)\right]\{\boldsymbol{\alpha}^{\omega}\}\ ; \qquad (3)$$

$$\left[\{\tilde{\mathbf{y}}_1\},...,\{\tilde{\mathbf{y}}_r\},\alpha_{r+1}^{y}\{\tilde{\mathbf{y}}_{r+1}\},...,\alpha_n^{y}\{\tilde{\mathbf{y}}_n\}\right]=\left[\tilde{\mathbf{Y}}\right]\{\boldsymbol{\alpha}^{y}\}, \qquad (4)$$

where $\{\boldsymbol{\alpha}^{\omega}\}^T=\left\{1,...,1,\alpha_{r+1}^{\omega},...,\alpha_n^{\omega}\right\}$, $\{\boldsymbol{\alpha}^{y}\}^T=\left\{1,...,1,\alpha_{r+1}^{y},...,\alpha_n^{y}\right\}$ are coefficients the values of which need to be specified. Initially, coefficients $\{\boldsymbol{\alpha}^{\omega}\}$ and $\{\boldsymbol{\alpha}^{y}\}$ have unity values.

The above presented modifications of the modal set preserve the physical essence of the unconstrained CS. It means, the modal frequencies corresponding to the rigid body modes are zeroes and the modal shape vectors remain orthogonal and express essentially the same modal shapes as before the modification. Also the total mass of the CS remains unchanged.

Now the modal frequency error minimization problem can be formally presented as

$$\min_{k_{Lj},\{\boldsymbol{\alpha}^{\omega}\},\{\boldsymbol{\alpha}^{y}\}} \quad \Psi=\sum_{i=r+1}^{\hat{N}}\left(\frac{\hat{\omega}_i-\hat{\omega}_{i0}}{\hat{\omega}_{i0}}\right)^2 \quad , \qquad (5)$$

where the penalty-type target function presents the cumulative modal frequency error of the structure, $\hat{\omega}_i$ - modal frequency of *i*-th mode of the structure, $\hat{\omega}_{i0}$ - its exact value known theoretically or obtained by using a highly refined finite element model. Number $\hat{N}$ of modes contributions of modal errors of which are included into function (5) can be selected freely. It means that namely these $\hat{N}$ lower modes will have minimized modal errors. We suggest $\hat{N}$ to be taken as 30-80% of the total number of modes of the sample domain. The larger is the individual CS, the larger number of modes can be expected to have small, say, 0.5-2% modal error values. Our numerical experiments demonstrate that by using CS of size 2x2 (CS_2x2) modal errors of about 30% of modes of the domain can be expected to be made lower than the above mentioned value. Meanwhile, the CS_5x5 allow to achieve minimized modal errors over almost 80% of modes. The thorough discussion on this is presented in section 4 of this paper.

The optimization is performed by assembling CS into *sample domains* of a shape similar to the shape of a CS provided that sufficiently large number of exact modal frequencies of the sample domain is known. As an example, for rectilinear and rectangular acoustic domains such modal frequencies are available analytically. In other cases a highly refined model of the sample domain can be used in order to obtain 'nearly exact' (say, <0.5% error) modal frequency values. The size of the sample domain practically is determined by a reasonable amount of calculations. Our numerical experiments demonstrate that often it is enough to perform optimization on a sample domain consisting of only several CS-s, and the optimized matrices of a single CS work well if a considerably larger structure is assembled. We can't present any theoretical proof of the validity of the approach, however, numerical experiments presented in [9] and in this work illustrate that it works.

4. NUMERICAL RESULTS

Fig.1 presents the results obtained by investigating the modal properties of quadrilateral acoustic sample domain 13x13 nodes assembled of quadrilateral elements (i.e. each CS is a 2x2 node quadrilateral element). Fig. 1a demonstrates the relative modal errors of the domain obtained by using lumped, consistent and generalized mass matrices. The modal error distribution for the three types of models is quite typical. Lumped matrices have a tendency to diminish the modal frequency values. On the contrary, consistent mass matrices give oversized values. The errors of the models employing the generalized mass matrices are always smaller, however the modal errors cannot be achieved to be close to zero over all modal frequency range. Fig.1b presents the results obtained by using optimized CS_2x2 as described in section 3. Number $\hat{N}$ in function (5) has been selected equal to ~30% of the total number of modes of the sample domain. Consequently, in Fig.1b ~30% of modes have modal errors less than 2%. The optimized models assembled of CS_2x2 do not demonstrate a marked difference in the modal error distribution when compared with the generalized mass matrix models. Much better results can be obtained by using larger CS-s. CS_5x5 has been optimized to form structures with minimal modal errors of more than 80% of the total amount of modes of the structure. The results of the optimization are presented in Fig.2. Fig.2a presents the modal errors of the sample domain 13x13 nodes assembled of optimized CS_5x5. The same CS_5x5 assembled to 29x29 node domain give modal errors presented in Fig.2b. In both cases all modal errors in the range of lower 80% modes of the structure do not exceed 1-2%. On the other hand, Fig.2a and b justify the assumption that the modal error distribution over the modal frequency range is nearly independent upon the size of the structure.

The performance of the optimized CS with respect to traditional elements is demonstrated in Fig.3 by analyzing the acoustic wave pulse propagating through a very roughly meshed domain. Excited by means of one sine pulse of normal velocity at the boundary excitation zone (Fig.3a) the circular wave front propagates and is reflected from the boundaries of the domain. The curves in Fig.3b demonstrate the propagating wave shapes obtained by using optimized mass matrix formulation and the reference wave shape obtained as a convergent solution in a densely meshed model. An excellent performance of the optimized model assembled of CS_5x5 is demonstrated where only ~5 elements used per wave pulse length enabled to get the shape of the wave in close resemblance to the reference wave shape.

Similar results can be observed by analyzing quadrilateral elastic domains. Fig.4 presents relative modal errors of lumped and consistent models. When combined to the generalized mass matrix with the weight of the lumped component kL=0.35 the relative error distribution is obtained less than 5% over all the modal frequency range. Therefore the computational wave propagation models obtained by using such generalized mass matrices are expected to have a very good performance. By performing the optimization process of the CS_3x3 the relative modal errors can be further diminished, Fig.4b. However, relative modal frequency errors of several modes cannot be made lower than 4%.

5. CONCLUSIONS

The research presents highly convergent 2D computational models for wave propagation simulations consisting of rectangular substructures. The computational models are assembled of *optimized component substructures* obtained by performing the optimization of the modal properties of a sample domain. Optimized component substructures assembled to larger domains demonstrate the same modal error distribution over the modal frequency range as has been obtained for the sample domain. The same component substructures can be used for assembling real computational domains of any size.

When compared with lumped, consistent or generalized mass matrices, the optimized component substructures produce significantly better results. However, the mass matrices of the optimized CS are non-diagonal. The obtained 2D models have very close-to-exact (less than 1-2% error) modal frequency values of more than ~80% of the total amount of modes of the structure and are able to present the propagating wave pulse shape by using only few nodal points per wavelength.

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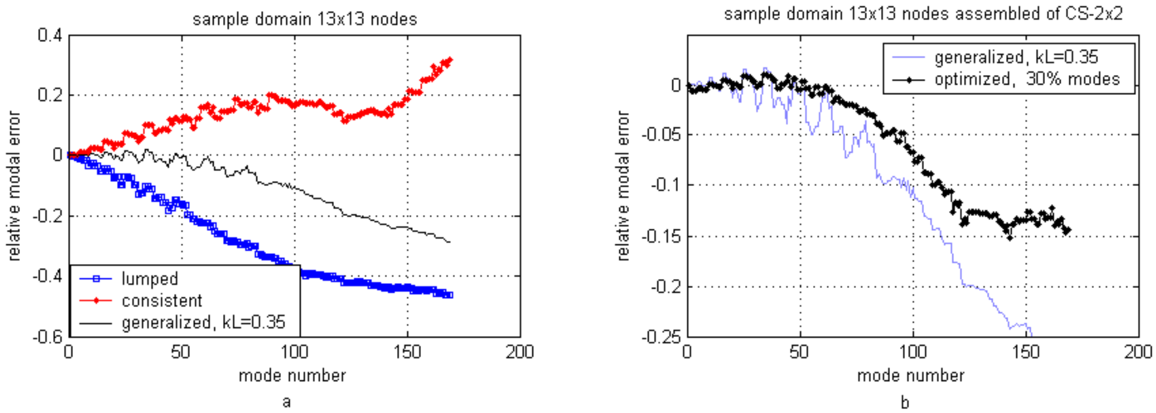

Fig. 1. Relative modal errors of the quadrilateral acoustic sample domain 13x13 nodes assembled of CS_2x2:
a- lumped, consistent and generalized mass matrices;
b- generalized mass matrices and optimized CS_2x2

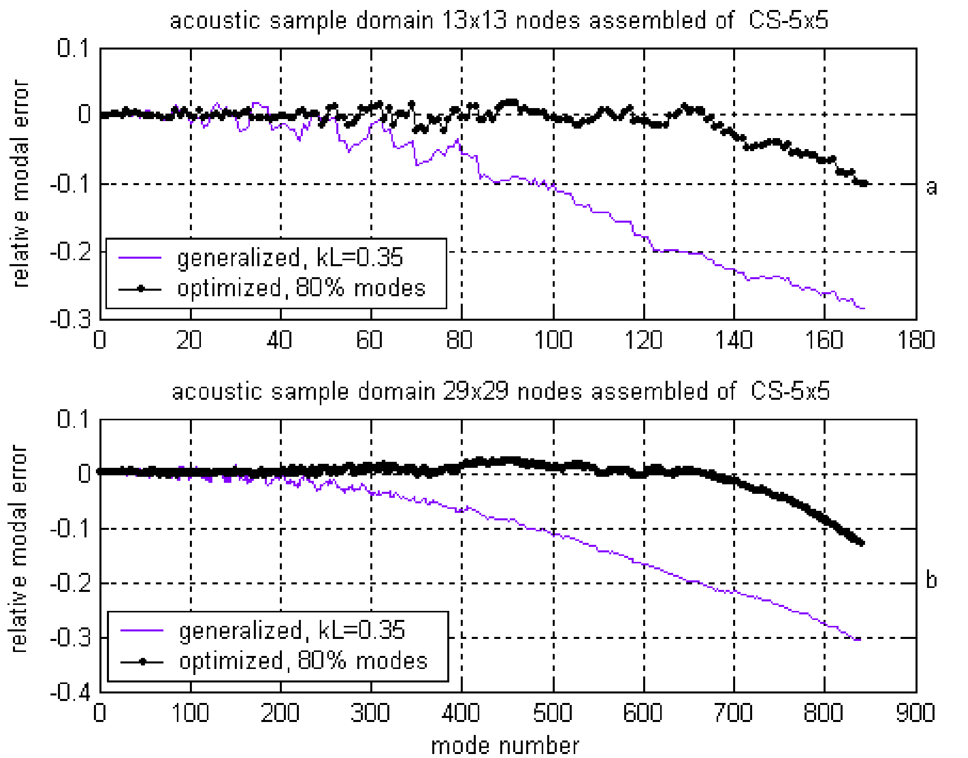

Fig. 2. Relative modal errors of the domains 13x13 (a) and 29x29 (b) nodes assembled of optimized CS_5x5

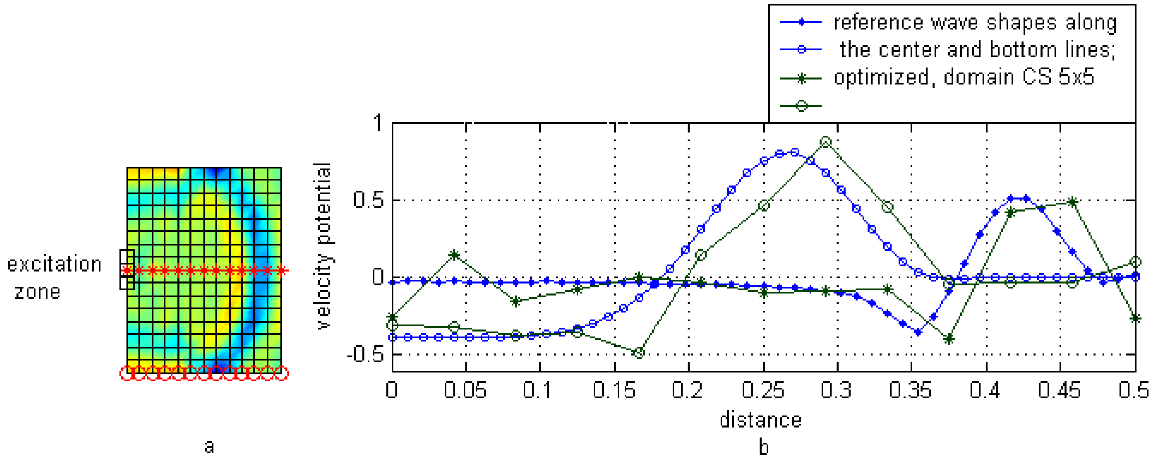

Fig. 3. Acoustic wave propagating in a roughly meshed (13x17) domain:
a – rough mesh and velocity potential contour plot at a given time point;
b – reference shape of the wave and the wave shape along the centerline of the model obtained by using the optimized and lumped mass matrix models CS 5x5.

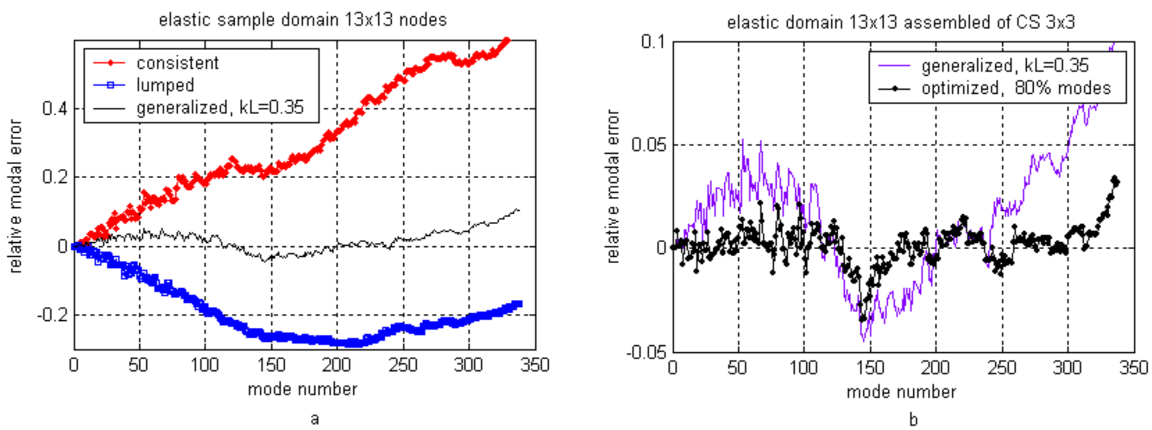

Fig. 4. Relative modal errors of quadrilateral elastic sample domain 13x13 nodes:
a –lumped, consistent and generalized mass matrices;
b –generalized mass matrices and optimized CS_3x3